Definition of a cut-off natural frequency for small bore pipework connections

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Summary

Vibration induced fatigue failures (VIFF) of pipework small bore connections (SBCs) due to turbulent flow excitation continue to occur in process piping systems, resulting in elevated safety risks and costly interruptions to normal operations. One of the primary causes of these failures is poor design.

In this paper, the concept of a cut-off natural frequency is defined, above which the probability of VIFF of SBCs is negligible. If this concept is incorporated into the design process for SBCs a decrease in the number of failures of SBCs due to vibration should be realized.

Introduction

When the valve and pipe components in an SBC assembly combine to produce a fundamental natural frequency that is low, the connection is susceptible to vibration with high levels of displacement. The high bending stresses associated with these high displacements, elevate the risk of VIFF. Examples of high risk SBCs are presented in Fig. 1.

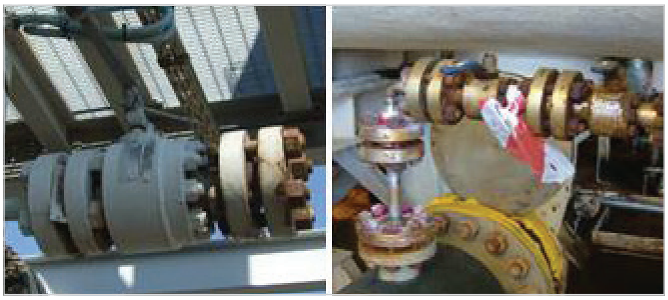

Fig. 1 – Examples of low natural frequency, high risk SBCs.

One of the primary design goals for SBCs should therefore be to achieve a fundamental natural frequency which is high enough to prevent high displacement vibration responses from being possible. This is very rarely considered in the existing design process.

Reasons for this include the lack of understanding of the importance of natural frequency to the vulnerability of the design and a lack of information on how high this target natural frequency must be when it is considered.

Fig. 2 illustrates the effect of increasing the fundamental response frequency of an SBC. It can be seen that for a given vibration response expressed in terms of velocity, the displacement at the tip of the SBC (free end) associated with the response significantly reduces as the frequency increases.

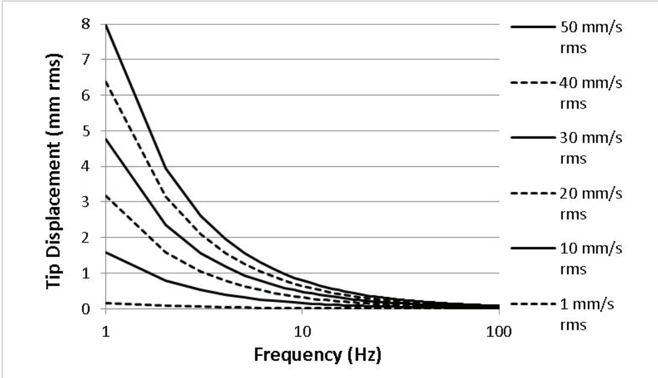

Fig. 2 – Displacement (mm rms) at tip (free end) of an SBC as a function of vibration magnitude (velocity) and response frequency.

In addition, a feature of turbulent flows in piping systems is that the available energy decays as the frequency increases. An example of this is shown in Fig. 3.

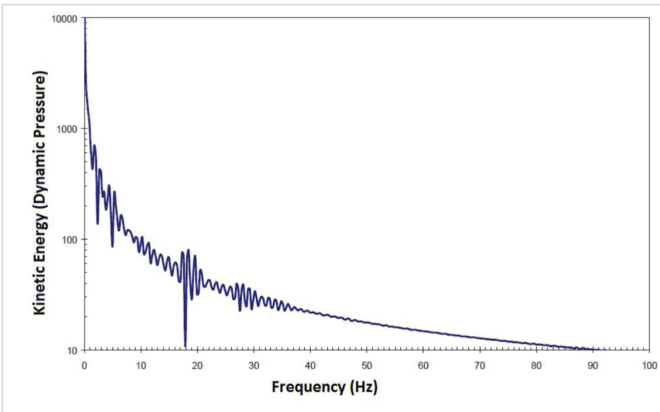

Fig. 3 – Turbulent energy distribution between 0 Hz and 100 Hz [1].

Using these two phenomena (the decreasing levels of displacement with increasing response frequency and the reduction in available energy with increasing frequency), it should be possible to define a fundamental natural frequency, above which the risk of VIFF from flow induced excitation is negligible. This frequency is defined as the cut-off natural frequency. This paper attempts to develop the concept of a cut-off natural frequency into a practical application that can be used during the design process. In this paper, 'low frequency' is defined as the frequency range below 30 Hz and 'high frequency' is defined as the frequency region between 30 Hz and 100 Hz (referencing figures 2 and 3) ▸

Nomenclature

Symbols

A	Pipe spool with wall thickness 1.65mm
A1	Pipe spool A with a 1kg end mass
A5	Pipe spool A with a 5kg end mass
B	Pipe spool with wall thickness 7.82 mm
B1	Pipe spool B with a 1kg end mass
B5	Pipe spool B with a 5kg end mass
D_o	Pipe outside diameter (mm)
E	Young's Modulus (Pa)
F	Force (N)
I	Second moment of area (mm4)
L	Spool length (mm)
a	Acceleration (mm/s2)
f	Excitation frequency (Hz)
fn	Natural frequency (Hz)
k	Stiffness (N/mm)
m	Effective mass (kg)
t	Spool wall thickness (mm)
x	Displacement (mm)
y/2	Maximum distance to centroid (mm)
δ	Damping ratio
σ	Bending stress (Pa)

Definitions

Geometry: A single combination of a mass and a spool length.
Geometry Range: The values of the maximum and minimum masses and spool lengths.
Geometry Set: All combinations of masses and spool lengths for a given section.
PSD: Power Spectral Density
SBC: Small Bore Connection
TR: Transmissibility Ratio
VIFF: Vibration Induced Fatigue Failure

Analysis Methodology

Assumptions and limitations

Several assumptions were made about the nature of the system in order to simplify the analysis.

Discrete excitation

This method only excites one frequency mode at a time and the contribution of higher order modes is not considered (higher order modes will generate lower levels of bending stress than the fundamental mode). A sinusoidal wave is applied at each frequency in turn and the response at the natural frequency is obtained for the results, as shown in Fig. 6.

Constant acceleration input

A constant acceleration of 0.52g is applied to the base of the SBC and is used to represent the vibration of the parent pipe which is excited by turbulent flow. The equivalent distribution of displacement magnitudes for this acceleration value is shown in Fig. 5. While this simulated energy profile is indicative of the actual one shown in Fig. 3, the absolute magnitudes are not equivalent leading to a degree of conservatism in the analysis.

System treated as a simple cantilever beam with a point mass at the end

The system is treated as a simple cantilever in order to establish a clear relationship between spool length and mass. This assumption excludes the more complex valve inertia effects from the analysis. Nor does it consider other types of SBC assemblies. The use of a flat, rigid base results in a uniform stress distribution.

Constant damping ratio

A constant structural damping ratio of 5% was assumed, ignoring any interdependence between damping, frequency and stiffness, thus allowing a constant transmissibility ratio to be used.

Geometry

The SBC natural frequency primarily depends on the stiffness of the spool and the end mass. Stiffness can be shown to be a function of spool length and the second moment of area as shown in Eq. 1, [2]

$$k = \frac{3EI}{L^3} \qquad (1)$$

Thus the geometry will be varied in order to analyse the bending stresses at different SBC natural frequencies. The spool diameter and wall thicknesses were selected from Trouvay & Cauvin [3] and represent real dimensions. The spool lengths and the end masses were chosen based on the experience of the authors and are representative of a class of SBCs found on oil and gas installations. These details are listed in Table 1. A nominal pipe diameter of 20mm (3/4" NB), with two wall thicknesses and two end masses was used. The spool length for each of the wall thickness/mass combinations was also varied to produce a [40 x 4] matrix of permutations.

Material: ASME B 36.10 M-1996
Pipe Nominal Size: 20mm (3/4" NB)
Seamless Carbon Steel
Density: 7850 kg/m^3
Yield Strength: 2.1x10^{11}Pa

Outer diameter, D_o (mm)	26.7			
Wall thickness, t (mm)	A		B	
	1.65		7.82	
Valve mass, m (kg)	A5	A1	B5	B1
	5	1	5	1
Spool length, L (mm)	[1000, 900, 800, 700, 600, 500, 400, 300, 200, 100]			

Table 1 – Configurations of the cantilever beams to be tested, [3].

The cantilever dimensions chosen are indicative of real SBCs but the limits of the range would not commonly be found on real piping systems. They serve primarily to demonstrate the relationships between various geometries. The chosen pipe wall thicknesses provide a wide range of stiffness and bending stress value for a given diameter

Since only one diameter value is used in the analysis, it is not referred to as a variable throughout this paper. Note that all analysis and discussion will be restricted to the results obtained using these geometries. Their limitations will be discussed in a separate subsection.

The worst and best case geometries of section A are shown in Fig. 4. The worst case is represented by a 5kg mass on a 1000mm spool and the best case is a 1kg mass on a 100mm spool.

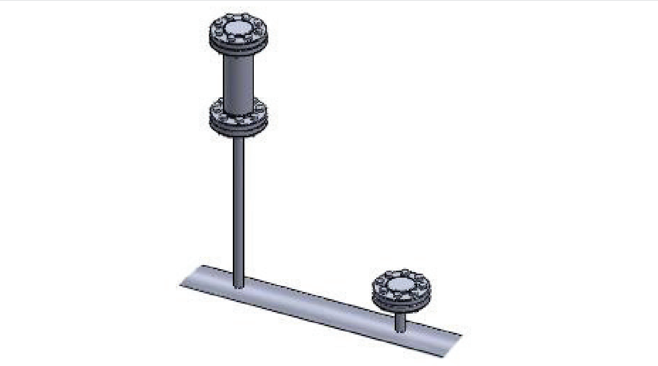

Fig. 4 – Worst (left) and best case geometries of section A.

Analysis Input

A constant acceleration is applied to all geometries in order to maintain a constant tip force for a given valve mass. The purpose of this is to evaluate the relationship between SBC geometry and the resultant bending stress as given by Eq. 2, [4]. From Eq. 3 it is clear that to keep the force constant the displacement must be varied proportionally to the inverse square of the frequency, as shown in Fig. 5.

$$\sigma = \frac{FLy}{2I} \qquad (2)$$

Here the force is given by Eq. 3.

$$F = ma = m(2\pi f)^2 x = kx \qquad (3)$$

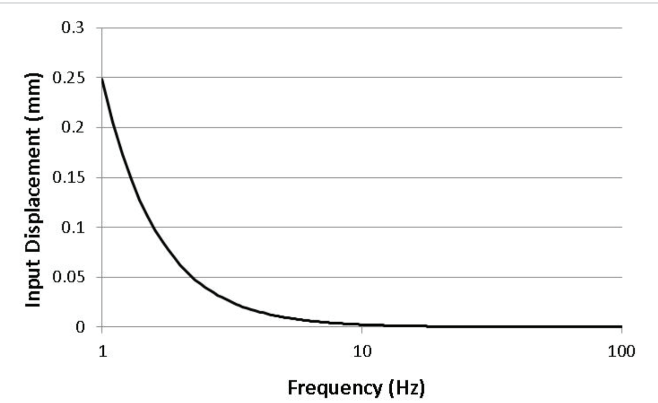

Fig. 5 – Input displacement as a function of frequency for a constant acceleration input.

Figure 5 shows that for a system with low frequency response, the available input energy is high, increasing the potential vibration magnitude and the likelihood of VIFF. The potential vibration magnitude decreases with increasing response frequency. An effective cut-off frequency will therefore be defined when the magnitude of the resonant response is below that required to produce fatigue damage.

The SBC geometries were assembled in ANSYS using pipe element type 288 and an acceleration of 0.52g was applied to the base of the SBC using a harmonic analysis. The resultant bending stresses were then plotted for each geometry against their respective resonant frequencies. This process highlighted which geometries produced the highest stresses and allowed the worst case scenario to be determined.

Displacement Transmissibility Ratio

The transmissibility ratio TR, can be calculated at any point in the frequency spectrum. Maximum TR occurs at resonance and is shown to be a factor of the damping ratio δ which remains constant for any geometry as shown in Eq. 4, [5]

$$TR_{max} = \sqrt{\frac{1}{4\delta^2} + 1} \qquad (4)$$

For δ=5%, TR=10.05. Multiplying any input displacement by this value gives the displacement output at resonance, from which the bending stress can be calculated. Thus by using the value of TR alone, the stress can be predicted for any geometry.

Results

Discrete excitation

Figure 6 shows the result of the 0.52g input applied to the two geometries shown in Fig 4. The bending stress recorded at the SBC connection to the main pipe is shown as a function of frequency, with the maximum stress level occurring at the fundamental natural frequency of the geometry.

Also shown, by the horizontal line, is the maximum permissible dynamic stress range of 17.5 MPa (peak to peak) for which remedial action is required as defined in the EI Guidelines [6] and which is based on the fatigue design S-N curve for an F2 class weld (using a safety factor of 2).

It can be seen that the fundamental natural frequency for the worst case geometry (solid line) is significantly lower than that of the best case (dashed line). It can also be seen that for an identical tip force, the bending stress for the worst case geometry is significantly higher than that of the best case.

In this example, the resulting bending stress level for the worst case geometry exceeds the acceptable stress limit and would lead to fatigue damage and probable VIFF.

The resulting stress level for the best case geometry remains below the acceptable stress limit and would not result in fatigue damage. The risk of VIFF for the best case geometry would therefore be negligible

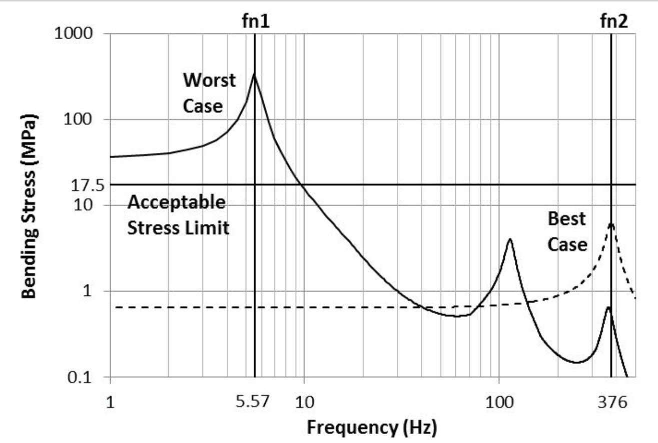

Fig. 6 – Peak bending stress of two different geometries.

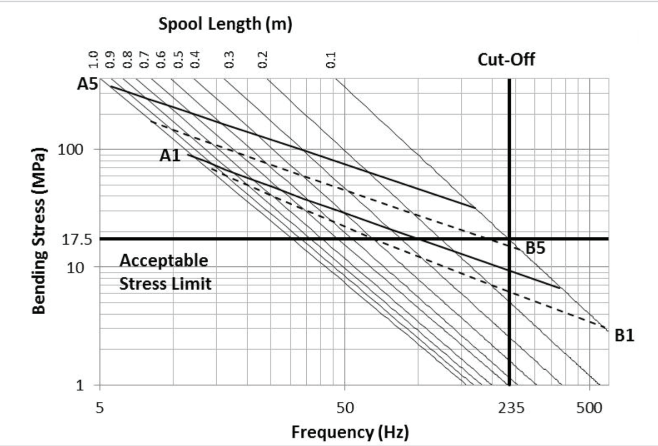

Fig. 7 – Bending stress for multiple geometries.

Bending stress plot for multiple geometries

The bending stresses for all permutations of variables in Table 1 are shown in Fig. 7. A detailed view of this figure is also provided in Annex A. Presenting the data on logarithmic axes establishes a linear representation of the relationship between the three variables. The vertical axis (ordinate) represents the bending stress magnitude for each geometry, whilst the horizontal axis (abscissa) represents the natural frequency of each geometry. The upper horizontal axis represents the spool lengths and is considered independently of the abscissa and ordinate axes. The horizontal line represents the EI Guidelines acceptable stress limit and the vertical line represents the cut-off frequency. Lines A1, A5, B1 and B5 represent their respective geometry sets.

Discussion

Bending stress and frequency

The bending stress results in Fig. 6 for the worst case and best case geometries clearly show the SBC with a higher natural frequency to have a lower bending stress. For the worst case geometry, Fig. 6 also shows that the stress associated with the fundamental natural frequency is significantly higher than the stress associated with the higher order modes; to the extent that the higher order modes can be discounted for the purposes of this investigation (the worst case results show 3 modes in the chosen bandwidth).

Because the TR remains constant but the displacement decays as a function of frequency (as shown in Fig. 2, Fig. 3 and Fig. 5), the magnitude of the bending stress decreases proportionately with the displacement. Given a high enough frequency, the stress level will drop below the EI Guidelines acceptable stress limit. Thus, the frequency at which the system vibrates with insufficient magnitude to cause fatigue damage can be determined. This point is defined as the cut-off frequency.

Stress results for multiple geometries

When the results are plotted for the geometry sets of both pipe cross-sections it is clear that one natural frequency can be associated with several geometries. Therefore the natural frequency alone cannot be relied upon in order to make an accurate assessment of the safety of a SBC since for a given frequency one geometry may fail, whilst another may pass. It is clear then, that in order to fully define the cut-off frequency; the bending stress magnitude of all geometries associated with a particular frequency must lie below the acceptable stress limit.

Some non-uniform behaviour is observed with the results obtained from section B. This is most likely due to the use of pipe element 288 in ANSYS which has a suggested thickness no greater than ¼ of the diameter. Use of a more suitable element may improve the uniformity.

Determining the cut-off frequency

The cut-off frequency can be clearly observed from Fig. 7. The worst case scenario is represented by the line **A5**. It is clear that the cross-over into the low risk (acceptable stress) zone occurs between the lines **A5** and **B5** at a spool length of 0.1m. This point defines the cut-off natural frequency. Below this frequency, the different geometries can have the same natural frequency, but different bending stress levels. Some of these geometries record unacceptable stress levels and some record acceptable stress levels. Above the cut-off frequency however, all geometries record acceptable stress levels. For the geometries that were investigated, the cut-off frequency is easily defined. Since all geometries are bound within the region defined by lines **A5** and **B1** and spool lengths 1m and 0.1m, the cut-off frequency is simply the first frequency at which no unacceptable stresses occur.

This means that only a specific set of geometries may have natural frequencies above this cut-off frequency, none of which experiences unacceptable levels of stress. It can also be said that cross section **A** is inherently unsafe, since not all of its masses have a safe configuration. Cross section **B** on the other hand can achieve a safe arrangement for all masses.

A significant outcome of this investigation is that a single natural frequency can be associated with many different stress levels depending on the geometry of the SBC. This could create difficulties at the design stage if the geometry range or the input is not accurately defined

Quadrant analysis

The analysis in section 4.3 is summarised in Table 2.

2nd quadrant	1st quadrant
All geometries fail.	In order to fully define a cut-off natural frequency no data can exist in this region.
3rd quadrant	**4th quadrant**
All geometries pass.	All geometries pass. Target area for design.

Table 2 – Characteristics of the four regions shown in Fig. 7.

Limitations

Energy

The results obtained in Fig. 7 are heavily dependent on the input energy. Had the input acceleration been of a different magnitude, the entire set of results would be shifted along the frequency (abscissa) axis and a new cut-off frequency would be defined. The cut-off frequency is also heavily dependent on the shape of the input energy profile. The approximation used in this analysis (as shown in Fig. 5) results in higher levels of stress response than would have been obtained using a more realistic energy distribution (as shown in Fig. 3). This would result in a significant downward shift in the cut-off frequency.

The distribution of turbulent excitation energy across a 100 Hz frequency band is shown in Fig. 3. As can be seen, most of the energy is concentrated below 30 Hz. It is clear that a discrete constant acceleration input is not an accurate representation of the energy distribution in Fig. 3 and therefore could overestimate the resultant stress levels. In order to refine this analysis, the input energy should be represented in the form of a PSD [7] and a more complex analysis technique should be investigated.

Geometry

At this stage, the cut-off frequency can only be clearly defined for a range of known dimensions. As stated in Tab. 2, for a cut-off frequency to be properly defined the 1st quadrant must not contain any data points. The cut-off frequency defined in this paper is valid only for the geometries investigated. In order to determine a cut-off frequency which can be applied globally for all combinations of SBC design, a significant amount of further analysis is required.

Conclusion

This investigation has determined that the concept of a cut-off natural frequency is a valid one and that it can be well defined for a given energy input and geometry range. This is ideal at the design stage since the energy input of the parent pipe can be accurately determined and dimensions of the mass, spool length and cross section are pre-determined by design constraints.

Therefore a range of SBC geometries can be quickly compared without the need for complex calculations.

This investigation has also demonstrated that a single natural frequency can be associated with a range of different SBC geometries, each with a unique and different bending stress. These findings are considered to be significant while recognizing the limitations of the geometries studied (it is unlikely in practice to achieve a natural frequency of 235 Hz).

The investigation should be extended to provide a more accurate representation of turbulent flow based excitation input in terms of both absolute magnitude and energy profile across the frequency range. The investigation should be extended to include a much wider range of SBC geometries in order to provide a more practical tool for application in the design process ■

References

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